HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate Trial Examination
Term 3 2024

STUDENT NUMBER: ______________________________

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- A NESA reference sheet is provided

Total Marks: 100

Section I – 10 marks (pages 3–6)

- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 - 29)

- Attempt Questions 11 - 34
- Allow about 2 hours and 45 minutes for this section

Q1 - 10	Q11 – 34	Total
/10	**/90**	**/100**

Outcomes assessed: MA 12 – 1, 3, 4, 5, 6, 7, 8, 10

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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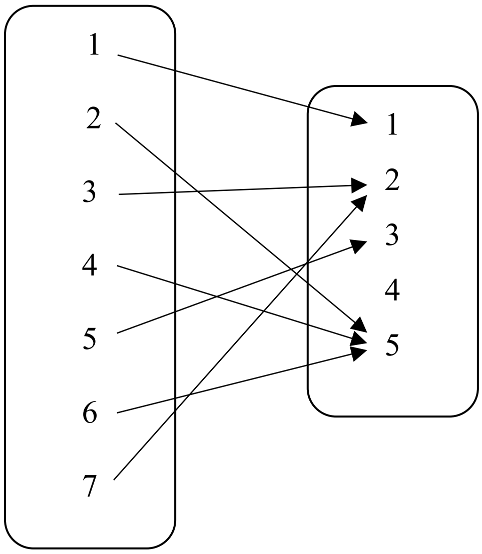
1 What type of relation is shown below?



A. One-to-One

B. One-to-Many

C. Many-to-One

D. Many-to-Many

2 If $f(x) = x^2 + 1$ and $g(x) = \sqrt{3 - x}$, then the domain for $f(x) \times g(x)$ is:

A. $(-\infty, \infty)$

B. $[3, \infty)$

C. $(-\infty, 3]$

D. $[-3, 3]$

3 What is the solution(s) of the equation $\log_2(x-3)+\log_2 x=2$?

A. $x=-1,\ x=4$

B. $x=-1$

C. $x=4$

D. $x=1,\ x=-4$

4 Consider the set of scores:

9 17 20 23 24 27 27 28 30 31 39 47

Which of the scores could be considered an outlier?

A. Only 9.

B. Only 47.

C. Both 9 and 47.

D. None of the scores.

5 What is the value of $\int_{-2}^{2}|x-1|\,dx$?

A. 0

B. 2

C. 4

D. 5

6 For what values of k does the equation $4(k+1)x^2+4kx+(k-2)=0$ have exactly two distinct roots?

A. $k>-2$

B. $k<-2$

C. $k\geq -2$

D. $k\leq -2$

7 The curve $y=f(x)$ has a maximum turning point at $(2,5)$.
For all values of x, $f(-x)=-f(x)$. Which of the following statements is true?

A. $(-2,5)$ is a minimum turning point of $y=f(x)$.

B. $(-2,-5)$ is a minimum turning point of $y=f(x)$.

C. $(-2,-5)$ is a maximum turning point of $y=f(x)$.

D. $(-2,5)$ is a maximum turning point of $y=f(x)$.

8 Suppose that $\int_{-3}^{8} f(x)\,dx = A$ and $\int_{2}^{8} f(x)\,dx = B$.

The value of $\int_{2}^{-3} \left(f(x)+2x\right)dx$ is:

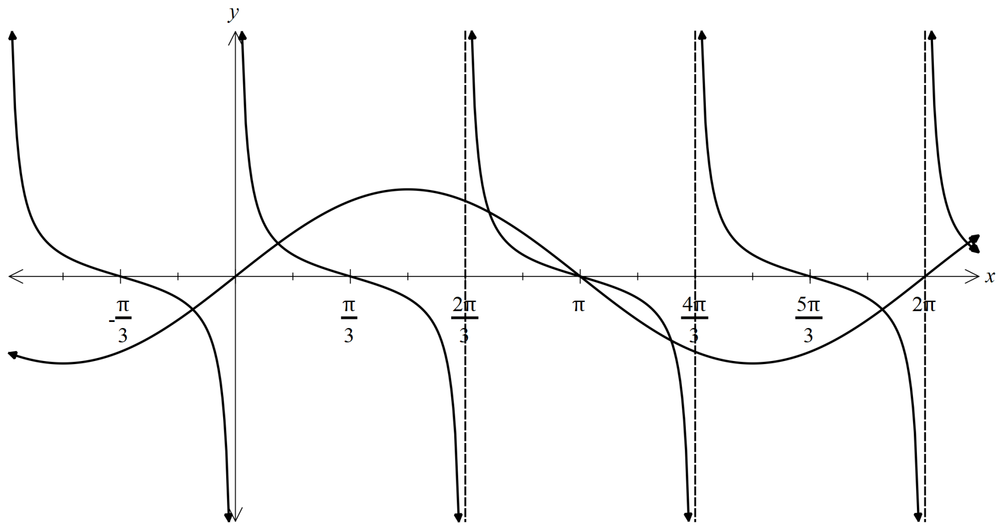
A. $A-B-5$

B. $B-A+5$

C. $A-B+5$

D. $B-A-5$

9 Below are the graphs of $y = 4\sin(x)$ and $y = \cot\left(\frac{3x}{2}\right)$.



How many solutions are there of $4\sin(x+\pi) - \cot\left(\frac{3x}{2}\right) = 0$ for $0 \le x \le 2\pi$?

A. 2

B. 3

C. 4

D. 5

10 What is the value of $\sum_{n=4}^{n} \ln 3^{2n}$?

A. $2n\ln 3$

B. $n(n+4)\ln 3$

C. $(n-4)(n+4)\ln 3$

D. $(n-3)(n+4)\ln 3$

End of Section I

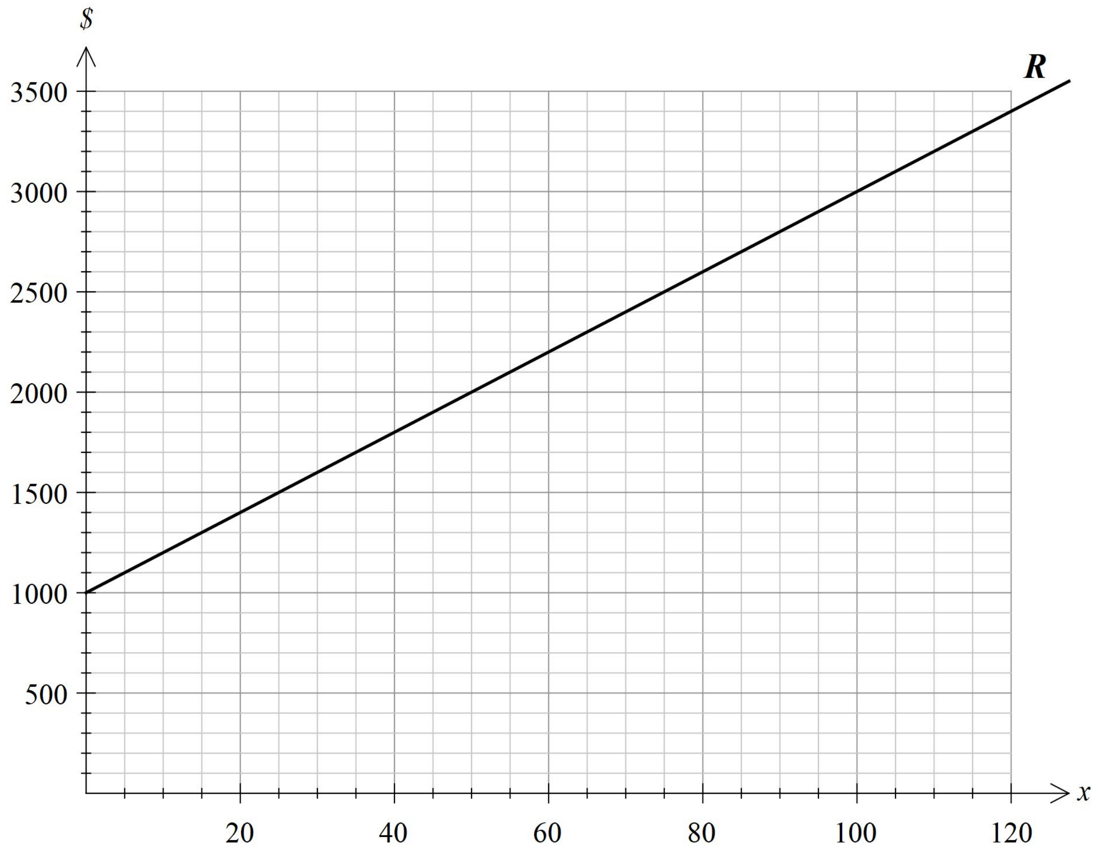
Section II

90 marks
Attempt questions 11 to 34
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (4 marks)

The incoming revenue for a catering company, R, is based on a \$1 000 hiring fee plus \$20 per person where x is the number of guests. The graph of the revenue model has been provided on the grid below.



(a) Write the equation for R in terms of x. **1**

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Question 11 continues on page 9

Question 11 (continued)

The cost for operating the catering company is modelled by $C = 1200 + 10x$, where C is the cost and x is the number of guests.

(b) On the grid on the previous page, draw the graph of this cost model and label it as C. **1**

(c) Using the graphs on the grid, or otherwise, find the value of x at the break-even point. **1**

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(d) Using the graphs, or otherwise, find the value of x when a profit of \$500 is made. **1**

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End of Question 11

Question 12 (3 marks)

A and B are two events such that $P(A) = 0 \cdot 3$, $P(B) = 0 \cdot 5$ and $P(A \mid B) = 0 \cdot 4$.

(a) Show that $P(A \cup B) = 0 \cdot 6$. **2**

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(b) Find in simplest exact form $P(\text{neither } A \text{ nor } B)$. **1**

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Question 13 (2 marks)

Evaluate $\displaystyle\int_{-1}^{3} \frac{5x^3 - 6x^2 + 3}{x^2}\, dx$. **2**

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Question 14 (4 marks)

The table shows the results of a survey of 300 people who have been vaccinated against a new virus.

(a) Complete the two-way table for the data collected. **2**

	Vaccinated	Not vaccinated	Totals
Infected	27		
Not infected		43	
Totals	137		300

(b) A person is selected at random from this group.

(i) Find the probability that the selected person is infected. **1**

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(ii) Find the probability that the selected person is not vaccinated, given that they are infected. **1**

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Question 15 (5 marks)

A car leaves a petrol station and drives $N30°W$ at 80 km/h. At the same time, a motor bike leaves the petrol station and travels due east at 60 km/h.

(a) Show that after one hour, the distance between the car and the motor bike is $20\sqrt{37}$ km. **2**

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(b) Find the bearing of the car from the motor bike. **3**

Give your answer correct to the nearest degree.

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Question 16 (4 marks)

Find the coordinates and nature of the stationary points on the curve $y = \dfrac{x^2}{x-2}$. **4**

Question 17 (2 marks)

Find $\int \frac{2x-1}{2x+1} dx$. **2**

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Question 18 (3 marks)

(a) Differentiate $y = x\log_e x$. **2**

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(b) Hence, find $\int \left(\log_e x + 1\right) dx$. **1**

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Question 19 (2 marks)

Which term of the arithmetic sequence 3, 16, 29, ... is 224? **2**

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Question 20 (2 marks)

The table below shows points on a continuous curve $y = f(x)$. **2**

x	3	3.2	3.4
$f(x)$	7.19	7.62	8.41

Use the trapezoidal rule to find the approximate value, correct to 2 decimal places, of

$$\int_{3}^{3.4} f(x)\ dx.$$

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Question 21 (6 marks)

Trout are released into a lake for the first time. The size of the population, P, of the trout can be modelled by the function $P = \dfrac{A}{1+15e^{-0.5t}}$, where t is the time in months after the trout are introduced into the lake.

(a) Initially, 250 trout were released into the pond. Show that $A = 4\ 000$. **1**

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(b) Find the range of the function P. **1**

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(c) At what rate is the population growing two months after the trout were released into the Lake? Give your answer correct to the nearest number of trout. **2**

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Question 21 continues on page 17

(d) How many months will it take for the population of trout to exceed 3 000? **2**

Give your answer as a whole number.

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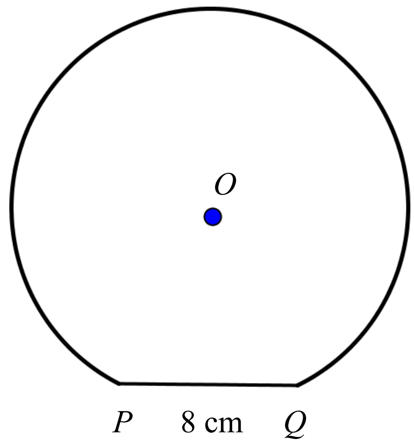
End of Question 21

Question 22 (4 marks)

A table tennis bat (without the handle attached) is in the shape of a major segment of a circle, with centre O and radius 8 cm. The length of the straight edge PQ is also 8 cm.

NOT TO SCALE



(a) Explain why $\angle POQ = \frac{\pi}{3}$. **1**

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(b) Find the exact area of the table tennis bat. **3**

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Question 23 (2 marks)

Prove that $(1+\sec x+\tan x)(1-\sec x+\tan x)=2\tan x$. **2**

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Question 24 (3 marks)

(a) Find the limiting sum of the geometric series $3+\frac{3}{\sqrt{3}+1}+\frac{3}{\left(\sqrt{3}+1\right)^2}+\ldots$. **2**

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(b) Explain why the geometric series $3+\frac{3}{\sqrt{3}-1}+\frac{3}{\left(\sqrt{3}-1\right)^2}+\ldots$ does not have a limiting sum. **1**

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Question 25 (5 marks)

A discrete random variable X with mean 4 has the probability distribution below for some constants a and p.

x	a	$2a$	$3a$	$4a$
$p(x)$	$4p$	$3p$	$2p$	p

(a) Find the values of a and p. **2**

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(b) Find $\text{Var}(X)$. **2**

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(c) Find $P(X \le 6 | X > 3)$. **1**

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Question 26 (4 marks)

The circle $x^2 + ax + y^2 + by + c = 0$ with a radius of 3 units, is shifted to the right by 1 unit. **4**

After the shifting, the circle has the centre at $(3,4)$. Find the value of a, b and c.

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Question 27 (3 marks)

Evaluate the sum of the geometric series $4 + 12 + 36 + \ ... \ + 26\ 244$. **3**

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Question 28 (6 marks)

A truck travels between two towns. The distance between the towns is B km.

The trucker has the following costs:

The hourly fuel costs are proportional to the square of the speed.

The hourly fuel cost is \$2000 when the speed is 40 km/h.

All other costs are \$4800 per hour. Let x be the speed in km/h and C be the total cost.

(a) Show that $C = B\left(\frac{5}{4}x + \frac{4800}{x}\right)$. **3**

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Question 28 continues on page 23

(b) What is the most economical speed? **3**

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End of Question 28

Question 29 (4 marks)

A teacher is interested in the relationship between how much time their class of 10 students spends on their phone with that of their laptop.

The teacher collected the results over the month of June and the average number of hours spent per day for each student is outlined in the table below.

STUDENT	A	B	C	D	E	F	G	H	I	J
Hours on phone per day (x)	7.6	7.0	8.9	3.0	3.0	7.5	2.1	1.3	5.8	6.2
Hours on laptop per day (y)	1.7	1.1	0.7	5.8	5.2	1.7	6.9	7.1	3.3	4.4

(a) Find the equation of the Least-Squares Regression Line. **2**

Give each coefficient to 3 decimal places.

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(b) Calculate Pearson's Correlation Coefficient, correct to 3 decimal places. **2**

What conclusion can be made about the time students spend on their phone compared to their laptops?

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Question 30 (6 marks)

A moving particle has an acceleration of a $\mathrm{m/s^2}$ at time t seconds $(t \geq 0)$.

The acceleration is given by the equation

$$a = 15 - 6t\,.$$

Initially the particle is at the origin and has a velocity of $v = -12$ m/s.

(a) Find the velocity, v, and displacement, x, in terms of t. **2**

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(b) Find the time(s) when the particle is at rest. **1**

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(c) Find the distance travelled during the first 4 seconds. **3**

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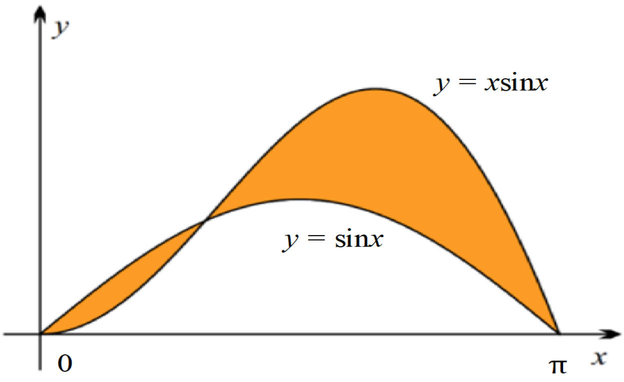
Question 31 (6 marks)

A sailing club has designed a new logo which is the shaded regions shown in the diagram below.

The shaded regions are bounded by $y = \sin x$ and $y = x\sin x$ in the interval $0 \le x \le \pi$.



NOT TO SCALE

(a) Find $\frac{d}{dx}(x\cos x)$ and hence show $\int x\sin x \, dx = \sin x - x\cos x + C$. **2**

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(b) Find the area of the logo. Give your answer in exact form. **4**

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Question 32 (3 marks)

Let $f(x) = e^{-kx} + 3x$, where k is a positive real number.

(a) Find, in terms of k, the x-coordinate of the stationary point of the graph of $y = f(x)$. **2**

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(b) State the values of k such that the x-coordinate of this stationary point is a positive real number. **1**

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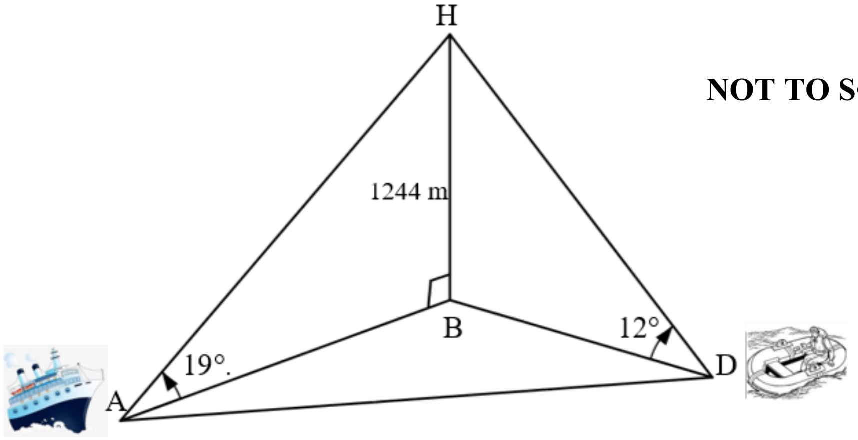
Question 33 (3 marks)

David is in a life raft, D, and Anna is in a cabin cruiser, A, searching for him. **3**

They are in contact by mobile telephone. David tells Anna that he can see the top of the mountain Mt Hope, H.

From David's position, the mountain has a bearing of 340° and the angle of elevation to the top of the mountain is 12°.

Anna can also see Mt Hope. From her position, the mountain has a bearing of 070° and the top of the mountain has an angle of elevation of 19°. The top of Mt Hope is 1 244 m above sea level.



Find the distance of the life raft from Anna's position.

Give your answer correct to two decimal places.

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Question 34 (4 marks)

On a particular day, the depth, D metres, of water in Blue Harbour is given by the function $D = 8 + 2\cos\left(\frac{4\pi}{25}t\right)$ for $0 \le t \le 24$, where t is the number of hours after midnight.

(a) Find the amplitude and period of the function. **2**

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(b) Find the first time when the water depth is 7 metres. **2**

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End of paper

HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate Trial Examination
Term 3 2024

STUDENT NUMBER: **Solutions**

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- A NESA reference sheet is provided

Total Marks: 100

Section I – 10 marks (pages 3–6)

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- Allow about 15 minutes for this section

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Q1 - 10	Q11 – 34	Total
/10	/90	/100

Outcomes assessed: MA 12 – 1, 3, 4, 5, 6, 7, 8, 10

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Mathematics Advanced

Year 12 Higher School Certificate Trial
Assessment 4 Term 3 2024

STUDENT NUMBER: SOLUTIONS

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1.	A ○	B ○	C ●	D ○
2.	A ○	B ○	C ●	D ○
3.	A ○	B ○	C ●	D ○
4.	A ○	B ●	C ○	D ○
5.	A ○	B ○	C ○	D ●
6.	A ●	B ○	C ○	D ○
7.	A ○	B ●	C ○	D ○
8.	A ○	B ●	C ○	D ○
9.	A ○	B ●	C ○	D ○
10.	A ○	B ○	C ○	D ●

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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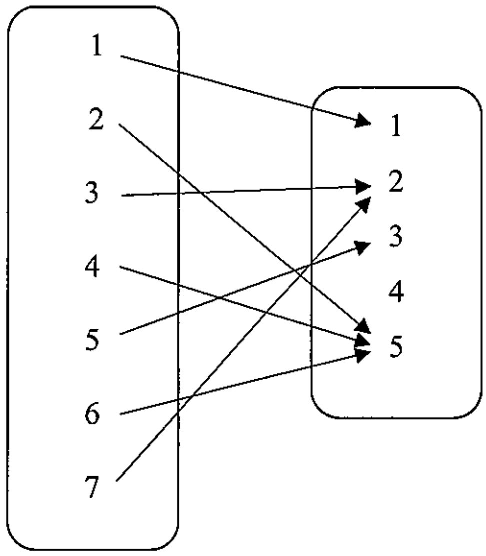
1 What type of relation is shown below?



A. One-to-One

B. One-to-Many

C. Many-to-One

D. Many-to-Many

2 If $f(x) = x^2 + 1$ and $g(x) = \sqrt{3-x}$, then the domain for $f(x) \times g(x)$ is:

A. $(-\infty, \infty)$

B. $[3, \infty)$

C. $(-\infty, 3]$

D. $[-3, 3]$

$f(x) \times g(x) = (x^2+1)\sqrt{3-x}$

$\therefore$ D: $3 - x \geq 0$

$-x \geq -3$

$x \leq 3$.

$(-\infty, 3]$

3 What is the solution(s) of the equation $\log_2(x-3)+\log_2 x = 2$?

A. $x=-1,\ x=4$

B. $x=-1$

(C.) $x=4$

D. $x=1,\ x=-4$

$\log_2[(x-3)\cdot x] = 2$
$x^2-3x = 2^2$
$x^2-3x-4=0$
$(x-4)(x+1)=0$
$x=4,\ x=-1$ but $x>3$
$\therefore x=4$ is only solution.

4 Consider the set of scores:

9 17 20 23 24 27 27 28 30 31 39 47

Which of the scores could be considered an outlier?

A. Only 9.

(B.) Only 47.

C. Both 9 and 47.

D. None of the scores.

$Q_1 = 21.5$
$Q_2 = 27$
$Q_3 = 30.5$
$IQR = Q_3 - Q_1$
$= 30.5 - 21.5 = 9$

Lower bound $= Q_1 - 1.5 \times IQR$
$= 21.5 - 1.5\times 9$
$= 8$

Upper Bound: $Q_3 + 1.5 \times IQR$
$= 30.5 + 1.5 \times 9$
$= 44$

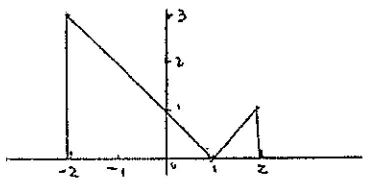
5 What is the value of $\int_{-2}^{2} |x-1|\,dx$?

A. 0

B. 2

C. 4

(D.) 5



$\int_{-2}^{2} |x-1|\,dx = \frac{1}{2}\times 3\times 3 + \frac{1}{2}\times 1\times 1$
$= \frac{9}{2} + \frac{1}{2}$
$= 5$

6 For what values of k does the equation $4(k+1)x^2+4kx+(k-2)=0$ have exactly two distinct roots?

(A.) $k>-2$

B. $k<-2$

C. $k\geq -2$

D. $k\leq -2$

$\Delta > 0$

$b^2 - 4ac > 0$

$(4k)^2 - 4(4(k+1))(k-2) > 0$

$16k^2 - 16(k+1)(k-2) > 0$

$16k^2 - 16(k^2 - k - 2) > 0$

$16k^2 - 16k^2 + 16k + 32 > 0$

$16k > -32$

$k > -2$

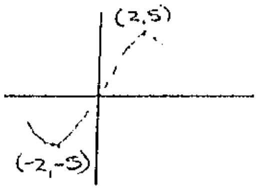
7 The curve $y=f(x)$ has a maximum turning point at $(2,5)$.
For all values of x, $f(-x)=-f(x)$. Which of the following statements is true?

A. $(-2,5)$ is a minimum turning point of $y=f(x)$.

(B.) $(-2,-5)$ is a minimum turning point of $y=f(x)$.

C. $(-2,-5)$ is a maximum turning point of $y=f(x)$.

D. $(-2,5)$ is a maximum turning point of $y=f(x)$.



8 Suppose that $\int_{-3}^{8} f(x)\,dx = A$ and $\int_{2}^{8} f(x)\,dx = B$.

The value of $\int_{2}^{-3} \left(f(x)+2x\right)dx$ is:

A. $A-B-5$

(B.) $B-A+5$

C. $A-B+5$

D. $B-A-5$

$\int_{-3}^{8} f(x)\,dx = \int_{-3}^{2} f(x)\,dx + \int_{2}^{8} f(x)\,dx$

$A = \int_{-3}^{2} f(x)\,dx + B$

$\int_{-3}^{2} f(x)\,dx = A - B$

$\therefore \int_{2}^{-3} f(x)\,dx = -(A-B)$
$= B - A$

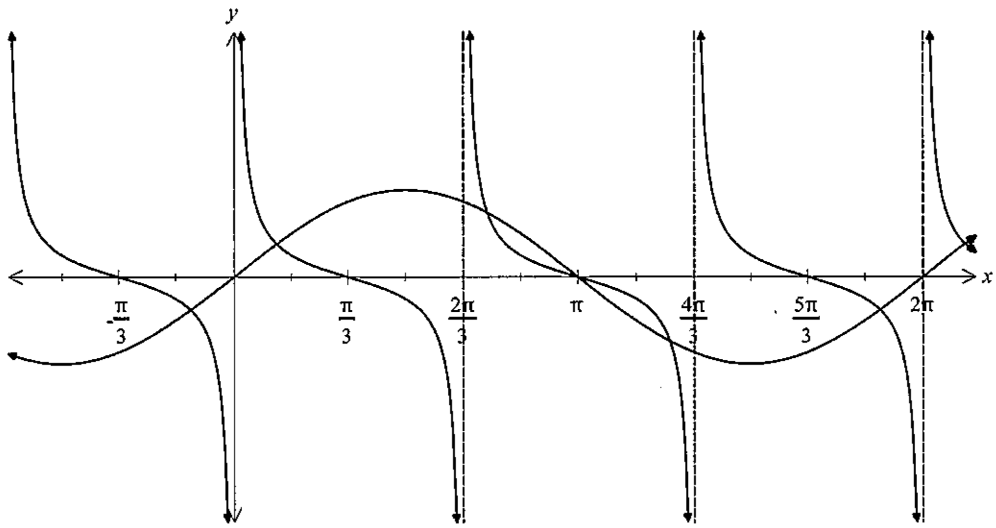
Now $\int_{2}^{-3} (f(x) + 2x)\,dx$

$= \int_{2}^{-3} f(x)\,dx + \int_{2}^{-3} 2x\,dx$

$= B - A + \left[x^2\right]_{2}^{-3} = B - A + 9 - 4$
$= B - A + 5$

- 5 -

9 Below are the graphs of $y = 4\sin(x)$ and $y = \cot\left(\frac{3x}{2}\right)$.



How many solutions are there of $4\sin(x+\pi) - \cot\left(\frac{3x}{2}\right) = 0$ for $0 \le x \le 2\pi$?

A. 2

(B.) 3

C. 4

D. 5

10 What is the value of $\sum_{n=4}^{n} \ln 3^{2n}$?

A. $2n\ln 3$

B. $n(n+4)\ln 3$

C. $(n-4)(n+4)\ln 3$

(D.) $(n-3)(n+4)\ln 3$

$\ln 3^8 + \ln 3^{10} + \ln 3^{12} \ldots + \ln 3^{2n}$
$= 8\ln 3 + 10\ln 3 + 12\ln 3 + \ldots + 2n\ln 3$
$= \ln 3(8+10+12+\ldots 2n)$
(Arithmetic Series
$a=8, d=2 \quad n = n-3, L = 2n$)
$= \ln 3[S_n] \quad \left(S_n = \frac{n}{2}(a+L)\right)$
$= \ln 3\left[\frac{n-3}{2}(8+2n)\right]$
$= \ln 3\left[\left(\frac{n-3}{2}\right)2(4+n)\right]$
$= (n-3)(n+4)\ln 3$.

End of Section I

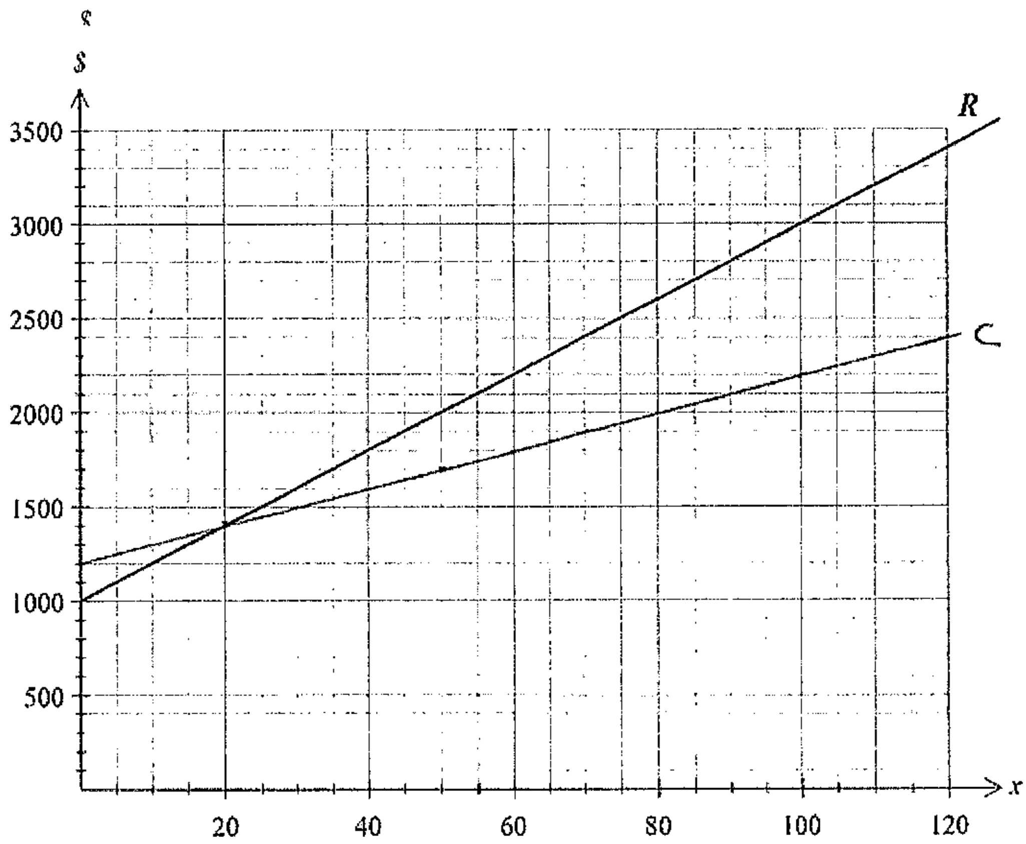
Section II

90 marks
Attempt questions 11 to 34
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (4 marks)

The incoming revenue for a catering company, R, is based on a \$1 000 hiring fee plus \$20 per person where x is the number of guests. The graph of the revenue model has been provided on the grid below.



(a) Write the equation for R in terms of x. **1**

$R = 1000 + 20x$ ....

....

....

Question 11 continues on page 9

Question 11 (continued)

The cost for operating the catering company is modelled by $C = 1200 + 10x$, where C is the cost and x is the number of guests.

(b) On the grid on the previous page, draw the graph of this cost model and label it as C. **1**

(c) Using the graphs on the grid, or otherwise, find the value of x at the break-even point. **1**

$R = C$

$1000 + 20x = 1200 + 10x$

$x = 20$ OR $10x = 200$

$x = 20$

(d) Using the graphs, or otherwise, find the value of x when a profit of $500 is made. **1**

$x = 70$

$R - C = P$

$1000 + 20x - (1200 + 10x) = 500$

$10x = 700$

$x = 70$

Some students misinterpretted the question as find the value of x when the revenue is $500 higher than the break-even point.

End of Question 11

Question 12 (3 marks)

A and B are two events such that $P(A) = 0\cdot3$, $P(B) = 0\cdot5$ and $P(A \mid B) = 0\cdot4$.

(a) Show that $P(A \cup B) = 0\cdot6$. **2**

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$0.4 = \dfrac{P(A \cap B)}{0.5}$

$P(A \cap B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.5 - 0.2$

$= 0.6$

Done well

(b) Find in simplest exact form $P(\text{neither } A \text{ nor } B)$. **1**

Lots of errors

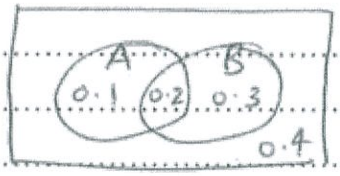
$P(\text{neither } A \text{ nor } B)$

$= 1 - P(A \cup B)$

$= 1 - 0.6$

$= 0.4$

or



Common error

$P(\bar{A}) = 0.7$

$P(\bar{B}) = 0.5$

$P(\bar{A} \cap \bar{B}) \neq 0.7 \times 0.5$

$P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) + P(\bar{B}) - P(\overline{A \cap B})$

$= 0.7 + 0.5 - (1 - P(A \cap B))$

$= 0.7 + 0.5 - (1 - 0.2)$

$= 1.2 - 0.8$

$= 0.4$

Question 13 (2 marks)

Evaluate $\displaystyle\int_{-1}^{3} \frac{5x^3 - 6x^2 + 3}{x^2}\, dx$. **2**

$$\int_{-1}^{3} \frac{5x^3 - 6x^2 + 3}{x^2}\, dx = \int_{-1}^{3} \frac{5x^3}{x^2} - \frac{6x^2}{x^2} + \frac{3}{x^2}\, dx$$

$$= \int_{-1}^{3} 5x - 6 + 3x^{-2}\, dx$$

$$= \left[\frac{5x^2}{2} - 6x + \frac{3x^{-1}}{-1}\right]_{-1}^{3}$$

$$= \left[\frac{5x^2}{2} - 6x - \frac{3}{x}\right]_{-1}^{3}$$

$$= \left[\frac{5(3)^2}{2} - 6(3) - \frac{3}{(3)}\right] - \left[\frac{5(-1)^2}{2} - 6(-1) - \frac{3}{(-1)}\right]$$

$$= \left[22\frac{1}{2} - 18 - 1\right] - \left[2\frac{1}{2} + 6 + 3\right]$$

$$= -8$$

← So, so! The first section was done very well. However many students didn't get the final answer correct

Question 14 (4 marks)

The table shows the results of a survey of 300 people who have been vaccinated against a new virus.

(a) Complete the two-way table for the data collected. **2**

	Vaccinated	Not vaccinated	Totals
Infected	27	120	147
Not infected	110	43	153
Totals	137	163	300

All parts done well

(b) A person is selected at random from this group.

(i) Find the probability that the selected person is infected. **1**

$P(\text{Infected}) = \frac{147}{300}$

(ii) Find the probability that the selected person is not vaccinated, given that they are infected. **1**

$P(\text{NotVaccinated}|\text{Infected}) = \frac{120}{147}$

Question 15 (5 marks)

A car leaves a petrol station and drives $N30°W$ at 80 km/h. At the same time, a motor bike leaves the petrol station and travels due east at 60 km/h.

(a) Show that after one hour, the distance between the car and the motor bike is $20\sqrt{37}$ km. **2**

$\angle COM = 120°$

$CM^2 = OC^2 + OM^2 - 2OC \times OM \cos\angle COM$

$CM^2 = 80^2 + 60^2 - 2\times 80 \times 60 \cos\angle 120°$

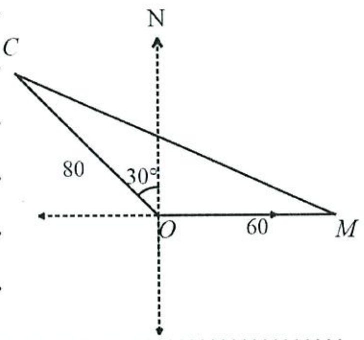
$CM^2 = 80^2 + 60^2 - 2\times 80\times 60\left(\frac{-1}{2}\right)$

$CM^2 = 14800$

$CM = \sqrt{14800}$ $(CM > 0)$

$CM = \sqrt{400\times 37}$

$= 20\sqrt{37}$ km



Mostly well done.
A few student didn't identify the correct direction of N30°W.

(b) Find the bearing of the car from the motor bike. **3**

Give your answer correct to the nearest degree.

By Sine rule,

$\frac{\sin\angle CMO}{OC} = \frac{\sin\angle COM}{CM}$

$\frac{\sin\angle CMO}{80} = \frac{\sin 120°}{20\sqrt{37}}$

$\sin\angle CMO = \frac{80\sin 120°}{20\sqrt{37}}$

$\angle CMO = \sin^{-1}\left(\frac{80\sin 120°}{20\sqrt{37}}\right)$

$= 34.71500...°$

$\approx 35°$

Since $\angle COM = 120°$, so $\angle CMO$ must be actute.

$\therefore$ the bearing of the car from the motor bike is approximately N55°W or 305°T.

Alternatively, use Cosine rule to find $\angle CMO$.

Mostly well done.

Question 16 (4 marks)

Find the coordinates and nature of the stationary points on the curve $y = \dfrac{x^2}{x-2}$. **4**

(16)

$y = \dfrac{x^2}{x-2}$, $x \neq 2$

stationary points ($y'=0$)

$$y' = \frac{(x-2).2x - x^2.1}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2}{(x-2)^2}$$

$$= \frac{x^2 - 4x}{(x-2)^2}$$

$$= \frac{x(x-4)}{(x-2)^2}$$

$= 0$ when $x = 0$ or when $x = 4$

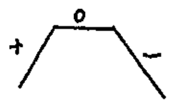
$\therefore y = 0$ $\quad$ $y = 8$

$\therefore$ stationary pts $(0,0)$ and $(4,8)$

Well done!

Testing Nature $(0,0)$

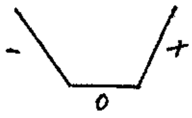
x	-1	0	1
y'	$5/9$	0	-3



$\therefore (0,0)$ is a maximum turning pt.

Testing Nature $(4,8)$

x	3	4	5
y'	-3	0	$5/9$



$\therefore (4,8)$ is a minimum turning pt.

Question 17 (2 marks)

Find $\int \frac{2x-1}{2x+1} dx$. **2**

$= \int \left(\frac{2x+1-2}{2x+1} \right) dx$

$= \int \left(1 - \frac{2}{2x+1} \right) dx$

$= x - \log_e |2x+1| + c$

This question was not done well.

Question 18 (3 marks)

(a) Differentiate $y = x \log_e x$. **2**

$y' = (\log_e x) \times 1 + x \times \frac{1}{x}$ Let $u = x$, $v = \log_e x$

$= \log_e x + 1$ $u' = 1$, $v' = \frac{1}{x}$

Done very well.

(b) Hence, find $\int (\log_e x + 1) dx$. **1**

$= x \log_e x + c$

Done well, due to the simplicity of the question a mark was not given if no +C.

Question 19 (2 marks)

Which term of the arithmetic sequence 3, 16, 29, ... is 224? **2**

$a = 3 \quad d = 13$

$T_n = a + (n-1)d$

$224 = 3 + (n-1) \times 13$

$13(n-1) = 221$

$n - 1 = 17$

$n = 18 \quad \therefore 224$ is the 18^{th} term.

Done very well ✓

Question 20 (2 marks)

The table below shows points on a continuous curve $y = f(x)$. **2**

x	3	3.2	3.4
$f(x)$	7.19	7.62	8.41

Use the trapezoidal rule to find the approximate value, correct to 2 decimal places, of

$$\int_3^{3.4} f(x)\, dx.$$

By trapezoidal rule,

$$\int_3^{3.4} f(x)\, dx \approx \frac{b-a}{2n}\left[f(3) + 2f(3.2) + f(3.4)\right]$$

$$\approx \frac{3.4-3}{2 \times 2}\left[(7.19) + 2(7.62) + (8.41)\right]$$

$$\approx 3.084$$

$$\approx 3.08 \quad \text{(to 2 dec. pl.)}$$

Some students need to revise the rule. n = number of subintervals not function values. Otherwise done well.

Question 21 (6 marks)

Trout are released into a lake for the first time. The size of the population, P, of the trout can be modelled by the function $P = \frac{A}{1+15e^{-0.5t}}$, where t is the time in months after the trout are introduced into the lake.

(a) Initially, 250 trout were released into the pond. Show that $A = 4\,000$. **1**

$t=0,\ P=250$

$250 = \frac{A}{1+15e^{-0.5(0)}}$

$250 = \frac{A}{16}$

$A = 4000$

well done.

(b) Find the range of the function P. **1**

as $t \to +\infty,\ e^{-0.5t} \to 0,\ P \to \frac{4000}{1+15(0)} = 4000$

since t is in months, $t \geq 0$,

At $t=0,\quad P=250$

$\therefore\ 250 \leq P \leq 4000$

Mostly well done. Some student didn't realise that initially $P=250$.

(c) At what rate is the population growing two months after the trout were released into the Lake? Give your answer correct to the nearest number of trout. **2**

$$\frac{dP}{dt} = \frac{d}{dt}4000\left(1+15e^{-0.5t}\right)^{-1}$$

$$= 4000(-1)\left(1+15e^{-0.5t}\right)^{-2}(-0.5)15e^{-0.5t} \qquad 1$$

$$= \frac{30000e^{-0.5t}}{\left(1+15e^{-0.5t}\right)^2}$$

$$t=2,\ \frac{dP}{dt} = \frac{30000e^{-0.5(2)}}{\left(1+15e^{-0.5(2)}\right)^2} \qquad 1$$

$$= \frac{30000}{e\left(1+15e^{-1}\right)^2}$$

$= 259.7601....$

≈ 259 trouts per months

$\therefore$ The population of trouts increases at approximately 259 trouts per months.

mostly well done. Some student couldn't find the correct derivative.

Question 21 continues on page 17

Question 21 (continued)

(d) How many months will it take for the population of trout to exceed 3 000? **2**

Give your answer as a whole number.

$$3000 = \frac{4000}{1+15e^{-0.5t}}$$

$$1+15e^{-0.5t} = \frac{4}{3}$$

$$15e^{-0.5t} = \frac{1}{3}$$

$$e^{-0.5t} = \frac{1}{45}$$

$$\ln e^{-0.5t} = \ln\frac{1}{45}$$

$$-0.5t = \ln\frac{1}{45}$$

$$t = \frac{\ln 45}{0.5}$$

$$= 7.613324....$$

$$\approx 8 \text{ months}$$

$\therefore$ it will take approximately 8 months for the population of trouts to exceed 3000.

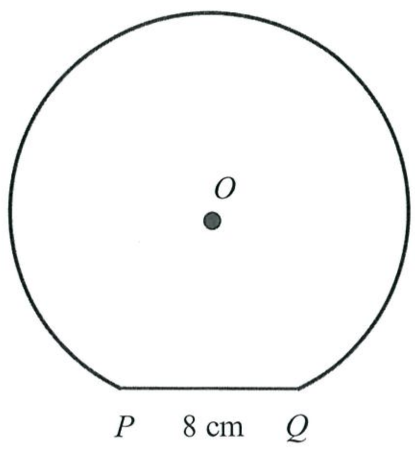
Mostly well done. Good to see majority of the students could solve exp. equations correctly.

End of Question 21

Question 22 (4 marks)

A table tennis bat (without the handle attached) is in the shape of a major segment of a circle, with centre O and radius 8 cm. The length of the straight edge PQ is also 8 cm.

NOT TO SCALE



(a) Explain why $\angle POQ = \frac{\pi}{3}$. **1**

$\angle POQ = \frac{\pi}{3}$ since $OP = OQ$ (radii of circle) $= 8$ cm

and $PQ = 8$ cm (given)

$\therefore \triangle POQ$ is equilateral

$\therefore$ all angles are $60^\circ = \frac{\pi}{3}^c$.

Well done! Some students didn't see the equilateral Δ immediately & worked out the angle using trigonometry.

(b) Find the exact area of the table tennis bat. **3**

Area of bat = Area of Circle − Area of minor segment

$= \pi \times 8^2 - \frac{1}{2} \times 8^2 \times \left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$ cm^2

$= 64\pi - 32\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$ cm^2

$= 64\pi - \frac{32\pi}{3} + 16\sqrt{3}$ cm^2

$= \frac{192\pi - 32\pi}{3} + 16\sqrt{3}$ cm^2

$= \frac{160\pi}{3} + 16\sqrt{3}$ cm^2

Poorly done! Many students used incorrect formulas.

Poorly done.
Many students could do the Q but did not set their working out well, especially when using trig identities.

Question 23 (2 marks)

Prove that $(1+\sec x+\tan x)(1-\sec x+\tan x)=2\tan x$. **2**

$$\text{LHS}=(1+\sec x+\tan x)(1-\sec x+\tan x)$$
$$=[(1+\tan x)+\sec x][(1+\tan x)-\sec x]$$
$$=(1+\tan x)^2-(\sec x)^2$$
$$=1+2\tan x+\tan^2 x-\sec^2 x$$
$$=(1+\tan^2 x)+2\tan x-\sec^2 x$$
$$=\sec^2 x+2\tan x-\sec^2 x$$
$$=2\tan x$$
$\therefore$ LHS = RHS (QED)

Needed to make use of trig identities and be clear with the substitution

common error
$(1+(\sec x+\tan x))(1-(\sec x+\tan x))$ should be −
and
$\left(1+\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)\left(1-\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)$
$=\left(1+\frac{1+\sin x}{\cos x}\right)\left(1-\frac{1+\sin x}{\cos x}\right)$ should be −

LHS $= 1-\sec x+\tan x+\sec x-\sec^2 x+\sec x\tan x+\tan x-\sec x\tan x+\tan^2 x$
$= 1+\tan x-\sec^2 x+\tan x+\tan^2 x$
$= (1+\tan^2 x)+2\tan x-\sec^2 x$
$= \sec^2 x+2\tan x-\sec^2 x$
$= 2\tan x$
$=$ RHS

Question 24 (3 marks)

(a) Find the limiting sum of the geometric series $3+\frac{3}{\sqrt{3}+1}+\frac{3}{(\sqrt{3}+1)^2}+\dots$. **2**

$a=3$ $r=\frac{1}{\sqrt{3}+1}$ $S=\frac{a}{1-r}$

$=\frac{3}{1-\frac{1}{\sqrt{3}+1}}$

$=\frac{3}{\frac{\sqrt{3}+1-1}{\sqrt{3}+1}}$

$=3\div\frac{\sqrt{3}}{\sqrt{3}+1}$

$=\frac{3\times(\sqrt{3}+1)}{\sqrt{3}}$

$=\frac{3\times(\sqrt{3}+1)}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=3+\sqrt{3}$

$=4.73205\ldots$ OK to give.

(b) Explain why the geometric series $3+\frac{3}{\sqrt{3}-1}+\frac{3}{(\sqrt{3}-1)^2}+\dots$ does not have a limiting sum. **1**

$a=3$ $r=\frac{1}{\sqrt{3}-1}=1.3660\ldots>1$

$|r|\nless 1$

$\therefore$ limiting sum does not exist.

Need to give the value of r and make comment that is correct.

Many students not quite understanding the concept of a limiting sum.

Question 25 (5 marks)

A discrete random variable X with mean 4 has the probability distribution below for some constants a and p.

x	a	$2a$	$3a$	$4a$
$p(x)$	$4p$	$3p$	$2p$	p

(a) Find the values of a and p. **2**

$$4p+3p+2p+p=1$$
$$10p=1$$
$$\therefore p=\frac{1}{10}$$

Given $E(X)=4$

i.e. $\sum xp(x)=4$

$$4ap+6ap+6ap+4ap=4$$
$$20ap=4$$
$$20a\left(\frac{1}{10}\right)=4$$
$$2a=4$$
$$\therefore a=2$$

Done very well

(b) Find $\text{Var}(X)$. **2**

$$\sum x^2p(x)=4a^2p+12a^2p+18a^2p+16a^2p$$
$$\therefore E(x^2)=50a^2p$$
$$Var(X)=E(x^2)-[E(x)]^2$$
$$Var(X)=50a^2p-(4)^2$$
$$=50(4)^2\left(\frac{1}{10}\right)-(4)^2$$
$$=20-16$$
$$=4$$

Different ways to answer this question. It was done reasonably well.

(c) Find $P(X\le 6|X>3)$. **1**

$$=\frac{3p+2p}{3P+2p+p}$$
$$=\frac{5p}{6p}$$
$$=\frac{5}{6}$$

OR

$$=\frac{\frac{3}{10}+\frac{2}{10}}{\frac{3}{10}+\frac{2}{10}+\frac{1}{10}}$$
$$=\frac{\frac{5}{10}}{\frac{6}{10}}$$
$$=\frac{5}{6}$$

Many students did not get this correct.

Question 26 (4 marks)

The circle $x^2+ax+y^2+by+c=0$ with a radius of 3 units, is shifted to the right by 1 unit. **4**

After the shifting, the circle has the centre at $(3,4)$. Find the value of a, b and c.

$x^2+ax+y^2+by+c=0$

$\left(x+\frac{a}{2}\right)^2-\frac{a^2}{4}+\left(y+\frac{b}{2}\right)^2-\frac{b^2}{4}+c=0$

$\left(x+\frac{a}{2}\right)^2+\left(y+\frac{b}{2}\right)^2=\frac{a^2}{4}+\frac{b^2}{4}-c$

shifted to the left by 1:

$\left(x-1+\frac{a}{2}\right)^2+\left(y+\frac{b}{2}\right)^2=\frac{a^2}{4}+\frac{b^2}{4}-c$

new centre at $(3,4)$:

$1-\frac{a}{2}=3,\quad \frac{-b}{2}=4$

$a=-4, b=-8$

radius is 3:

$\frac{a^2}{4}+\frac{b^2}{4}-c=3^2$

$\frac{4^2}{4}+\frac{(-8)^2}{4}-c=9$

$c=4+16-9$

$=11$

alternatively, ~~setse~~ original centre $(2,4)$

$(x-2)^2+(y-4)^2=9$

$x^2-4x+4+y^2-8x+16-9=0$

$x^2-4x+y^2-8y+11=0$

$\therefore a=-4,\ b=-8,\ c=11.$

Mostly well done. Some students couldn't complete the square properly.

Question 27 (3 marks)

Evaluate the sum of the geometric series $4+12+36+\ \ldots\ +26\ 244$. **3**

$a=4\quad r=3$

$ar^{n-1}=T_n$

$4.3^{n-1}=26\ 244$

$3^{n-1}=6561$

$3^{n-1}=3^8$

Equating powers

$n-1=8$

$\therefore n=9$

or $n-1=\log_3 6561$

$=\frac{\ln 6561}{\ln 3}$

$=8$

$\therefore n=9$

$S_n=\frac{a(r^n-1)}{r-1}$

$=\frac{4(3^9-1)}{3-1}$

$=2(3^9-1)$

$=39\ 364$

This question was done reasonably well.

& Some students need to review the rule.

Question 28 (6 marks)

A truck travels between two towns. The distance between the towns is B km.

The trucker has the following costs:

The hourly fuel costs are proportional to the square of the speed.

The hourly fuel cost is \$2000 when the speed is 40 km/h.

All other costs are \$4800 per hour. Let x be the speed in km/h and C be the total cost.

Lots of problems showing the result.

(a) Show that $C = B\left(\frac{5}{4}x + \frac{4800}{x}\right)$. **3**

Hourly fuel cost $\propto x^2$

$= kx^2$

$2000 = k \times 40^2$

$\frac{2000}{1600} = k$

$k = 1{\cdot}25 = 1\frac{1}{4} = \frac{5}{4}$ 1. for showing working to find $k = \frac{5}{4}$

Total cost per hour = hourly fuel cost + other costs per hour

Hourly Cost $= \frac{5}{4}x^2 + 4800$ 1.

Total cost, $C = \left(\frac{5}{4}x^2 + 4800\right) \times$ number of hours

Speed = $\frac{\text{Distance}}{\text{Time}}$

Time = $\frac{\text{Distance}}{\text{Speed}} = \frac{B}{x}$ 1. for finding time $= \frac{B}{x}$

$C = \left(\frac{5}{4}x^2 + 4800\right) \times \frac{B}{x}$

$C = B\left(\frac{5}{4}x + \frac{4800}{x}\right)$

Question 28 continues on page 23

Question 28 (continued)

(b) What is the most economical speed? **3**

$C = B\left(\frac{5}{4}x + \frac{4800}{x}\right)$

$\frac{dC}{dx} = B\left(\frac{5}{4} + -4800x^{-2}\right)$

$= B\left(\frac{5}{4} - \frac{4800}{x^2}\right)$

Stationary points occur when $\frac{dC}{dx} = 0$

$\frac{5}{4} - \frac{4800}{x^2} = 0$ 1 for correct diff and equating to zero

$\frac{5}{4} = \frac{4800}{x^2}$

$5x^2 = 4800 \times 4$

$x^2 = \frac{4800 \times 4}{5}$

$x^2 = 3840$

$x = \pm 61.96773354$ 1. for solving

but $x \geq 0$ since speed is positive

$\therefore x = 61.97$

$\frac{d^2C}{dx^2} = B\left(0 - (-2)\,4800x^{-3}\right)$

$= B \times \frac{9600}{x^3}$

When $x = +61.9677$ ($x = \sqrt{3840}$) 1 for justifying minimum cost

$\frac{d^2C}{dx^2} = B \times \frac{9600}{61.9677^3} > 0$ since $B > 0$ as B is a distance

$\therefore$ minimum cost i.e. most economical speed

when $x = \sqrt{3840}$

$x \doteqdot 61.97$ km/h

Many students neglected to show that the cost was in fact a minimum $\therefore$ giving the most economical speed

End of Question 28

Question 29 (4 marks)

A teacher is interested in the relationship between how much time their class of 10 students spends on their phone with that of their laptop.

The teacher collected the results over the month of June and the average number of hours spent per day for each student is outlined in the table below.

STUDENT	A	B	C	D	E	F	G	H	I	J
Hours on phone per day (x)	7.6	7.0	8.9	3.0	3.0	7.5	2.1	1.3	5.8	6.2
Hours on laptop per day (y)	1.7	1.1	0.7	5.8	5.2	1.7	6.9	7.1	3.3	4.4

(a) Find the equation of the Least-Squares Regression Line. **2**

Give each coefficient to 3 decimal places.

Well done!

$A = 8.375821... \approx 8.376 \; (3dp)$

$B = -0.875156... \approx -0.875 \; (3dp)$

$y = 8.376 - 0.875x$

(b) Calculate Pearson's Correlation Coefficient, correct to 3 decimal places. **2**

What conclusion can be made about the time students spend on their phone compared to their laptops?

Well done!

$r = -0.96229... \approx -0.962 \; (3dp)$

This is a strong negative correlation which means the more time a person spends on the phone, the less time they spend on their laptop.

Question 30 (6 marks)

A moving particle has an acceleration of a m/s^2 at time t seconds $(t \geq 0)$.

The acceleration is given by the equation

$$a = 15 - 6t.$$

Initially the particle is at the origin and has a velocity of $v = -12$ m/s.

(a) Find the velocity, v, and displacement, x, in terms of t. **2**

$t = 0,\ x = 0,\ v = -12$

$a = 15 - 6t$

$v = 15t - 3t^2 + c$

$-12 = 0 - 0 + c$

$c = -12$

$\therefore v = 15t - 3t^2 - 12$

$x = \dfrac{15t^2}{2} - t^3 - 12t + d$

$0 = 0 - 0 - 0 + d$

$d = 0$

$x = \dfrac{15t^2}{2} - t^3 - 12t$

Well done. A few did not make the change from the errata sheet.

(b) Find the time(s) when the particle is at rest. **1**

Rest when $v = 0$

$-3t^2 + 15t - 12 = 0$

$t^2 - 5t + 4 = 0$

$(t-1)(t-4) = 0$

$\therefore$ At rest when $t = 1, t = 4$ seconds

Done well.

(c) Find the distance travelled during the first 4 seconds. **3**

When $t = 0,\ x = 0$

When $t = 1,\ x = -5.5$

When $t = 4,\ x = 8$

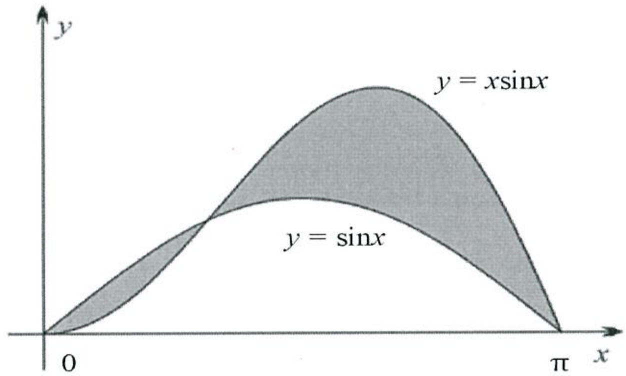
Total distanced travelled in the 4 seconds $= 5.5 + 5.5 + 8 = 19$ metres

Some answered this question giving only displacement for t=4. (1) mark for this work.

Question 31 (6 marks)

A sailing club has designed a new logo which is the shaded regions shown in the diagram below.

The shaded regions are bounded by $y=\sin x$ and $y=x\sin x$ in the interval $0\le x\le\pi$.



NOT TO SCALE

(a) Find $\frac{d}{dx}(x\cos x)$ and hence show $\int x\sin x\,dx=\sin x-x\cos x+C$. **2**

$$\frac{dy}{dx}(x\cos x)=\cos x-x\sin x$$ 1

$$\int(\cos x-x\sin x)\,dx=x\cos x$$

$$\int\cos x\,dx-\int x\sin x\,dx=x\cos x$$ 1

$$\int x\sin x\,dx=\int\cos x\,dx-x\cos x$$

$$=\sin x-x\cos x+C$$

Mostly well done. Some students need to revise the topic on reverse chain rule.

(b) Find the area of the logo. Give your answer in exact form. **4**

Find the intersection point:

$\sin x=x\sin x$

$\sin x(1-x)=0$

$\sin x=0, x=1$

$x=0,\pi,1\quad(0\le x\le\pi)$ 1

Since the intersection point is between 0 and π,

$\int_0^1(\sin x-x\sin x)\,dx+\int_1^\pi(x\sin x-\sin x)\,dx$

$=\left[-\cos x-(\sin x-x\cos x)\right]_0^1+\left[(\sin x-x\cos x)--\cos x\right]_1^\pi$ 1

$=(-\cos1-\sin1+1\cos1)-(-\cos0-\sin0+0\cos0)+\left[(\sin\pi-\pi\cos\pi+\cos\pi)-(\sin1-1\cos1+\cos1)\right]$

$=-\sin1+1+(\pi-1-\sin1)$

$=\pi-2\sin1$ unit^2 1

Mostly well done. Some calculation mistakes.

Question 32 (3 marks)

Let $f(x) = e^{-kx} + 3x$, where k is a positive real number.

(a) Find, in terms of k, the x-coordinate of the stationary point of the graph of $y = f(x)$. **2**

Stationary points occur where $f'(x) = 0$

$f'(x) = -k \cdot e^{-kx} + 3 = 0$ — 1 for correct differentiation and equating to zero

$k e^{-kx} = 3$

$e^{-kx} = \frac{3}{k}$

$\ln e^{-kx} = \ln \frac{3}{k}$

$-kx = \ln \frac{3}{k}$

$x = \frac{\ln \frac{3}{k}}{-k}$ or $\frac{-\ln \frac{k}{3}}{-k}$ — 1 for correct expression

$= \frac{\ln \frac{k}{3}}{k}$

Done well by most students

(b) State the values of k such that the x-coordinate of this stationary point is a positive real number. **1**

Poorly done

$x > 0$ when $\ln \frac{3}{k} < 0$

$\ln 3 - \ln k < 0$

$\ln 3 < \ln k$

$\ln k > \ln 3$

$k > 3$

or $\ln \frac{3}{k}$ is only defined when $\frac{3}{k} > 0$ and is negative when $\frac{3}{k} < 1$

$\therefore 0 < \frac{3}{k} < 1$

$\frac{k}{3} > 1$

$k > 3$

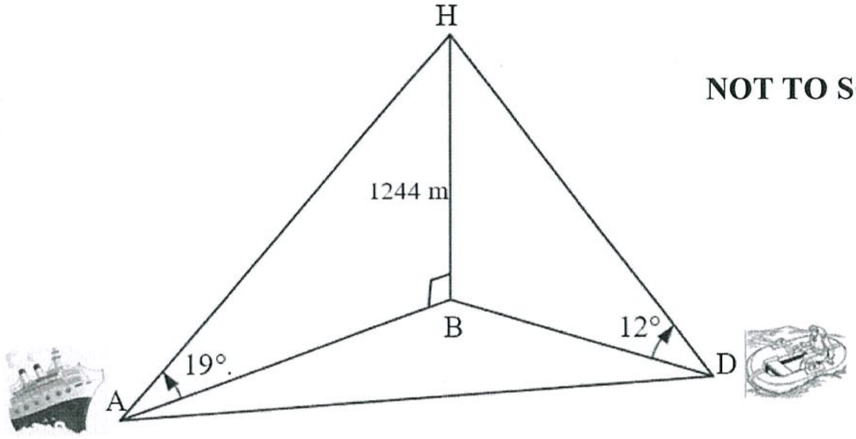
Question 33 (3 marks)

David is in a life raft, D, and Anna is in a cabin cruiser, A, searching for him. **3**

They are in contact by mobile telephone. David tells Anna that he can see the top of the mountain Mt Hope, H.

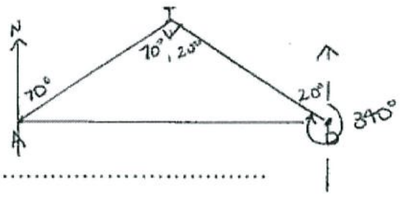
From David's position, the mountain has a bearing of 340° and the angle of elevation to the top of the mountain is 12°.

Anna can also see Mt Hope. From her position, the mountain has a bearing of 070° and the top of the mountain has an angle of elevation of 19°. The top of Mt Hope is 1 244 m above sea level.



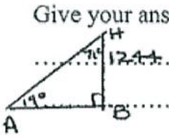
Find the distance of the life raft from Anna's position.

Give your answer correct to two decimal places.



Find the distance of the life raft from Anna's position.

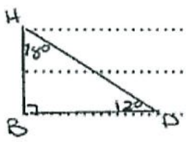
Give your answer correct to two decimal places.



$\tan 71^\circ = \frac{AB}{1244}$

$\therefore AB = 1244 \times \tan 71^\circ$

Generally well done!



$\tan 78^\circ = \frac{BD}{1244}$

$\therefore BD = 1244 \tan 78^\circ$

In ΔABD: $AD^2 = AB^2 + BD^2$

$= (1244 \tan 71^\circ)^2 + (1244 \tan 78^\circ)^2$

$\doteqdot 47305057.68$

$\therefore AD = \sqrt{47305057.68}$ $AD > 0$

$\doteqdot 6877.867\ldots$

$\doteqdot 6877.87$ m (2dp)

Question 34 (4 marks)

On a particular day, the depth, D metres, of water in Blue Harbour is given by the function $D = 8 + 2\cos\left(\frac{4\pi}{25}t\right)$ for $0 \le t \le 24$, where t is the number of hours after midnight.

(a) Find the amplitude and period of the function. **2**

amplitude = 2

period $= \frac{2\pi}{n}$

$= 2\pi \div \frac{4\pi}{25}$

$= 2\pi \times \frac{25}{4\pi}$

$= \frac{25}{2}$ ∴ period $= 12\frac{1}{2}$ hours

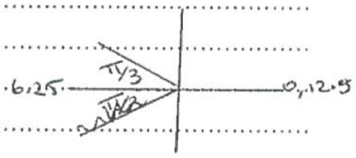
Well done! Some students become confused with the division of the fraction for the period

(b) Find the first time when the water depth is 7 metres. **2**

∴ $8 + 2\cos\left(\frac{4\pi}{25}t\right) = 7$

$2\cos\left(\frac{4\pi}{25}t\right) = -1$

$\cos\left(\frac{4\pi}{25}t\right) = -\frac{1}{2}$



∴ $\frac{4\pi}{25}t = 6.25 + \frac{\pi}{3}$

∴ $t = \frac{2\pi}{3} \div \frac{4\pi}{25}$

$t = 4$h 10mi

∴ 4:10 am

The first section was done well. However, many students didn't write the time – they just left the answer in hours.

End of paper